**Item Analysis by the Hierarchical Generalized Linear Model**

Author(s): Akihito Kamata

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This paper provides an example of an explicit alternative formulation of a multilevel item response model, both as a two-level model and, as an extension, a three-level model.

HGLM is an extension of the generalized linear model (GLM) (McCullagh & Nelder, 1989) to hierarchical data that enables HLM to deal with models having non-normal errors.

Four perspectives on multilevel formulation of IRT models are represented as background information：

* This first perspective is characterized by the treatment of person ability parameters as random parameters in an IRT model, a treatment originally intended to facilitate MMLE item parameter estimation.
* A second perspective on multilevel formulation of IRT models is represented by the multiple-group IRT model, which assumes individuals are grouped by a common characteristic, such as ethnic group or school attended.
* The third perspective is the decomposition of an item parameter into more than one parameter in IRT modeling.
* The fourth perspective on multilevel formulation of IRT models relates to attempts to investigate rater effects in rating scales, such as performance assessments.
* First, the two-level item analysis model is formulated and shown to be algebraically equivalent to the Rasch model.

The two-level item analysis model is formulated following the GLM framework：

Level-1 is the item-model

The linear predictor model of a sampling distribution of item responses is considered to be the level-1 model.

*log(pij/(1- pij))= ηij=β0j+β1j X1ij + β2j X2ij +…+βkj Xkij*

*= β0j+*

*Pij*: the probability that j gets item i correct

*i*: item (1,…, k )

*j*: person (1,…, n )

*q*: dummy variable q=1 when q=i or q=0

*β0j* : intercept

*βqj*: coefficient associated with *X****qij***

Level-2 is the person-model, in which β0j is assumed to be a random effect across persons.

*β0j=γ00 +μ0j*

*β1j=γ10*

*…*

*β (k-1)j=γ(k-1)0*

*μ0j:*random component of *β0j*

when level-l and level-2 models are combined, the linear predictor model is as follows:

When i=q, the equation is algebraically equivalent to Rasch Model.

* Next, a two-level item analysis model with person-characteristic variables is presented, offering a means of analyzing the effects of the person variables alone. Level-1 is the same, while a predictor variable is added in level-2.

*β0j=γ00 +γ01W1j +…+ γ0pWpj+ μ0j\**

*β1j=γ10*

*…*

*β (k-1)j=γ(k-1)0*

Wsj are person-level predictor measures for predictor s and person j.

* Lastly, a three-level item analysis is presented that can provide estimates of group-level abilities as well as person-level abilities, quantify the variation of person-characteristic variable effects across groups, and reveal any interaction effect between a group-characteristic variable and a person-characteristic variable.

For example, a new level that represents school is added to the model.

Level-1——item-level model：

*i=1, .. , k-1*

*j=1, .. ,n*

*m=1, .. ,r*

:the *q*th dummy variable for the *i*th item for person *j* in school *m*

: the effect of the reference item

: the effect of the *q*th item compared to the reference item

The level-2 models for the parameters assumed to be constant across people are person-level models.

*=*

*=*

*…*

*=*

how much personj in school m is deviated from the mean of within school *m*

: The variance of

the effect of the *i*th item (for *i = q*) in school *m*, assuming the effect of the *k*th item is zero.

level-3: school-level model could show that item effects are constant across schools

For school m, we have

*…*

is a fixed component of *,*  is a random component*,* is variance of the *.*

The combined model is

As an illustrative analysis, data from the Third International Mathematics and Science Studies (TIMSS) are analyzed by the three-level model.17 multiple-choice items done by 1,130 students from 68 schools are included in the analysis.

This result presents evidence that the effect of studying science at home on students' science literacy test scores depends on teachers' experience.

Comments:

The paper offers a full perspective for me to understand the HLM. The principal and procedure from one step to another is very clear which is a good material for new learners to know more about this kind of models. It also brings me some ideas about my own research proposal.

At first, I have no idea why the Q dummy variable is needed in the model. Though my thought may be wrong, I think it is for the item level analysis. With this variable we can tell the difference among items or person, even school.

I think I need to do some real analysis to use the model instead of just reading.