As already mentioned, this observation about ratios could be applied to the method of magnitude estimation or, more directly, to ordinal judgments of relative magnitude. One magnitude estimation procedure involves the experimenter assigning some number, say 100, to a standard stimulus, and then instructing the subject to assign numbers to other stimuli in such a way that the ratios of these numbers to 100 express the magnitude of the stimuli to that of the standard (relative to some specified attribute). Order relations between such numerical ratios could easily be used to test cancellation conditions on the relative magnitudes (or ratios) of the stimuli. Alternatively, subjects could be instructed to order stimulus ratios directly. Either way the hypothesis that human judgments about some attribute are quantitative could be tested.

CONCLUSION

As must be obvious by now, the science of psychology is still a long way from developing procedures that may legitimately be described as measurement procedures. However, despite Pythagorian cries to the contrary, this does not impugn psychology's status as a science. Its status as a science rests solely on its use of the methods of observation, hypothesis, and deduction. It needs no more to be given that status and in this it is no different to any other science. Psychological measurement will only ever be a reality if some psychological variables are quantitative and that issue is an empirical one and must be settled by experimental research. What has been considered in this book are the kinds of experimental evidence that will support (or, alternatively, falsify) the hypothesis that some psychological variable is quantitative. The theory of conjoint measurement provides a method whereby such evidence may be collected.

In this chapter a number of developments that will enhance the quest for measurement in psychology have been looked at. The primary need is for the development of techniques of stimulus control. This is necessary not only for any application of conjoint measurement theory but also for that special application, the construction of standard sequences. It is this, more than anything else, that will establish psychological measurement. In the absence of such sequences legitimate doubts must remain.

The other developments considered relate to extending the theory and applications of conjoint measurement. This is necessary if the full range of quantitative theories used within psychology are to be brought within its scope.

Beyond these needs there lies another: the need to struggle against the mistaken belief that psychological measurement already exists. This illusion is based on the false views of measurement prevalent within psychology. That entrenched illusion is now the major obstacle in the way of those who would sooner test the reality, than accept the myth of psychological measurement.

Appendix 1

The Theory of Order

Quantification in psychology is distinguished from that in physics by the extent to which it has been attempted from a purely ordinal base. Order, of course, is one of the characteristics of quantity. Within psychology it has sometimes been assumed that where there is order, there must also be quantity. Thus psychologists have postulated a number of quantitative theories to explain ordinal effects such as the ones considered in this book. Although this approach has not yet been resoundingly successful in establishing psychological measurement on a sure footing, it has given rise to some interesting theories, some widely used techniques (especially multidimensional scaling), and one important discovery, the theory of conjoint measurement. Through this theory it has been discovered that measurement can be achieved upon a purely ordinal base.

THE CONCEPT OF ORDER

An order on a class of entities is a relation that lines them up so that each entity is earlier or later than each other. Consider, for example, the points on the line in Fig. 1. They are ordered from left to right as A, B, C, D, E, F, G, and H. Any order is a relational system and there are a number of different ways that it can be described. One of the simplest is to characterize the order in terms of the relations holding between the pairs of entities. One such relation in Fig. 1 is that

FIG. 1. An illustration of an order: the eight points, *A, B, C, D, E, F, G,* and *H* are ordered from left to right along a line.

TABLE 1
Some Properties of Binary Relations

A relation, R, is *transitive* upon a class if and only if, for every x, y and z in that class, if $R(x,y)$ and $R(y,z)$ then $R(x,z)$.
e.g., the relation of being an ancestor of upon the class of all people is transitive.

A relation, R, is *intransitive* upon a class if and only if, for every x, y and z in that class, if $R(x,y)$ and $R(y,z)$ then not $R(x,z)$.
e.g., the relation of being the mother of upon the class of all people is intransitive.

A relation, R, is *symmetric* upon a class if and only if, for every x and y in that class, if $R(x,y)$ then $R(y,x)$.
e.g., the relation of being a sibling of upon the class of all people is symmetric.

A relation, R, is *asymmetric* upon a class if and only if, for every x and y in that class, if $R(x,y)$ then not $R(y,x)$.
e.g., the relation of being the father of upon the class of all people is asymmetric.

A relation, R, is *antisymmetric* upon a class if and only if, for every x and y in that class, if $R(x,y)$ and $R(y,x)$ then $x=y$ (i.e., x is identical to y).
e.g., the relation of being at least as great as upon the real numbers is antisymmetric.

A relation, R, is *reflexive* upon a class if and only if, for every x in that class, $R(x,x)$.
e.g., the relation of being as tall as upon the class of all people is reflexive.

A relation, R, is *irreflexive* upon a class if and only if, for every x in that class, not $R(x,x)$.
e.g., the relation of being the brother of upon the class of all people is irreflexive.

A relation, R, is *strongly connected* upon a class if and only if, for every x and y in that class, either $R(x,y)$ or $R(y,x)$.
e.g., the relation of being at least as great as upon the natural numbers is strongly connected.

A relation, R, is *connected* upon a class if and only if, for every x and y in that class such that $x \neq y$, either $R(x,y)$ or $R(y,x)$.
e.g., the relation of being less than upon the natural numbers is connected.

of being to the left of: *A* is to the left of *B; B* is to the left of *C;* and so on. This relation orders all eight points and it does so because it possesses certain properties. It is transitive, asymmetric and connected (these and other important properties of binary relations are defined in Table 1). Any relation that is transitive, asymmetric, and connected on a particular class of entities orders the members of that class.

A logically equivalent description of this relation of being to the left of is to characterize it as being transitive, irreflexive, and connected. This is because any transitive and asymmetric relation is also transitive and irreflexive, and vice versa. So any relation that is transitive, irreflexive, and connected on a class also orders the members of that class.

In chapter 3 the fact that the values of a quantitative variable are ordered was expressed, not by referring to a relation that was transitive, asymmetric (or irreflexive), and connected, but by referring to one that was transitive, antisymmetric, and strongly connected (in that case the relation of being at least as great as). This again, is simply an alternative way of describing an order. In the case of Fig. 1 the order could also be described in terms of the relation of not being to the right of, a relation that is transitive, antisymmetric, and strongly connected. That is, a class is ordered by a transitive, asymmetric, and connected relation if and only if it is ordered by a transitive, antisymmetric, and strongly connected relation. To take another example, the real numbers are ordered by both $>$ (which is transitive, asymmetric, and connected) and $\geqslant$ (which is transitive, antisymmetric, and strongly connected).

Despite the fact that the structure described by reference to each kind of relation is the same (viz., an order) the relations themselves are given different names. Transitive, antisymmetric, and strongly connected relations are known as *simple orders* and transitive, asymmetric (or irreflexive), and connected relations are known as *strict simple orders* (cf. Suppes, 1957 & Burington, 1965).

Orders are highly prized natural structures because of their simplicity. So it is not surprising that some structures that are not orders, should be assessed in terms of their proximity to an order. Some that have been identified are weak orders, partial orders, and quasi–orders.

Weak orders may be illustrated by extending Fig. 1 so that more than one point occurs at some of the positions going from left to right. This is done in Fig. 2. Such a system of points is not an order because neither of the two relations considered (being to the left of and not being to the right of) possess the necessary properties. Being to the left of is no longer connected, and not being to the right of is no longer antisymmetric. However, the relation of not being to the right of

FIG. 2. An illustration of a weak order: the relation of not being to the right of on this set of points is both transitive and strongly connected.

is transitive, and strongly connected, and it is these two properties which characterize a weak order.

The great beauty of a weak order is illustrated in Fig. 3. If the points in Fig. 2 are grouped according to their position going from left to right, then the resulting set of groups is an order. That is, any two points, x and y, belong to the same *equivalence class* if and only if x is not to the right of y and y is not to the right of x. These equivalence classes are exhaustive (every point belongs to one) and discrete (every point belongs to only one), and they stand in the following relation to each other: group X is not to the right of group Y, if and only if each member of X is not to the right of any member of Y. This relation of one group not being to the right of another is transitive, antisymmetric, and strongly connected. Hence, it constitutes an order.

This reduction of a weak order to an order may be expressed formally as follows. Let R be any transitive and strongly connected relation on a class (i.e., R is a weak order). A relation, E, definable in terms of R must then exist as follows: for any x and y in the class,

$$E(x,y) \text{ if and only if } R(x,y) \text{ and } R(y,x).$$

E is an equivalence relation (i.e., it is transitive, asymmetric, and reflexive) and pairs of objects standing in this relation belong to the same equivalence class. Objects *x's* equivalence class, $[x]$, is the set of all things y such that $E(x,y)$. Equivalence classes stand in a relation R^* to one another as follows:

$$R^*([x], [y]) \text{ if and only if } R(x,y).$$

R^* upon the class of equivalence classes is a simple order. Because a weak order always reduces to a simple order in this fashion it is often called a *preorder*. From the logical point of view, having a weak order is as good as having an order.

This is not generally so with partial orders. The relation of being to the left of upon the points of Fig. 2 is transitive and asymmetric (though it is not connected) and so is, what is called, a *strict partial order*. Of course, in this particular example, the equivalence relation and corresponding equivalence classes may also be defined by reference to this relation. The equivalence relation holds between points that are indifferent to each other with respect to the relation of being to the left of (e.g., points B and C). That is, indifference with respect to this relation is an equivalence relation, but in general this is not so for transitive and asymmetric relations.

Let R be any transitive and asymmetric relation. Indifference with respect to $R(I)$ is as follows: for any x and y in the class,

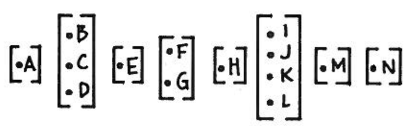

FIG. 3. When the points in Fig. 2 are grouped together the resulting groups constitute an order.

$$I(x,y) \text{ if and only if not } R(x,y) \text{ and not } R(y,x).$$

It does not follow from the properties of R that such an indifference relation must be an equivalence relation. Consider a family tree. The relation of being an ancestor of is transitive, asymmetric and so, a strict partial order. A hypothetical family tree is illustrated in Fig. 4. In that figure, an arrow from X to Y indicates that X is an ancestor of Y. Now B and G are indifferent with respect to this relation (neither is the ancestor of the other) as are G and E, but B and E are not. Hence, the indifference relation is not transitive. Of course as we have seen, in special cases it may be transitive, but in general it need not be. When it is, the strict partial order is equivalent to a weak order. When it is not, the structure may still closely approximate an order, as is seen in the cases of the *semiorder* and *interval order*.

Semiorders and interval orders are special cases of strict partial orders. Suppose that a relation, R, is irreflexive upon a class of objects. Then R is a semiorder if and only if, for any x, y, z, and w in the class

(i) if $R(x,y)$ and $R(z,w)$, then $R(x,w)$ or $R(z,y)$ and
(ii) if $R(x,y)$ and $R(y,z)$, then $R(x,w)$ or $R(w,z)$.

R is an interval order if and only if (i) is true. An irreflexive relation that satisfies (i) must also be transitive (to see this let $y = z$), and any transitive and irreflexive relation is asymmetric. So if R is an interval order (and if it is a semiorder it is an interval order), then it must be both transitive and asymmetric. Hence, semiorders and interval orders are strict partial orders.

A simple example of a semiorder might be an ordering of, say, a set of weights by a person. Since small differences between weights are not discernable, pairs of indistinguishable but different weights may be included in the set relative to any judge. Suppose that the relation of one weight being judged greater than another (R) is irreflexive and transitive. Also, suppose that there exists a weight increment, e, such that for any weights, x and y, in the set, $R(x,y)$ if and only if $x \geqslant y + e$. (e is the just noticeable difference (jnd) between weights). Then, of course, (i) and (ii) are true and so R is a semiorder on the weights. On the other hand, if the magnitude of the jnd is relative to the weight involved, so that each weight, x, has its own particular jnd, e_x, then $R(x,y)$ if and only if $x \geqslant y + e_y$.

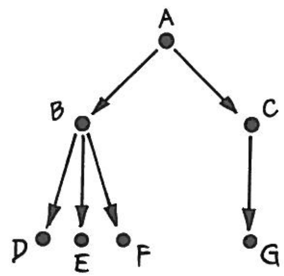

FIG. 4. A hypothetical family tree illustrating intransitive indifference.

In this case (i) is true but (ii) false and so R is an interval order. Fishburn (1970) and Roberts (1976) give more details on these kinds of structures.

To each strict partial order there corresponds a partial order and conversely. A partial order is a relation that is transitive, antisymmetric, and reflexive. It is simply an alternative characterization of the same kind of structure as a strict partial order. If a relation, R, is a strict partial order than the disjunction of R and the identity relation constitutes a partial order. For example, the composite relation of being an ancestor of or identical with on the family tree in Fig. 4 is a partial order.

Table 2 summarizes these ordinal structures. Within each row the structures characterized are equivalent. They differ only with respect to the relation selected to describe the structure. Row A contains the orders proper; row B, those structures that may be reduced to an order on equivalence classes; and row C, those structures that only approximate an order.

Psychologists have taken an interest in orders largely because of the association between orders and quantity. Obviously, quantity entails order. This, together with the fact that some physical quantities were initially identified only ordinally (e.g., temperature) has encouraged psychologists to treat order as a sign of quantity. This treatment is slightly presumptuous for two reasons. First, order alone does not entail quantity. Second, ordinal structures do not necessarily require quantitative explanation. Some of the conditions under which ordinal structures do require quantitative explanation are discussed in chapter 4.

A PSYCHOLOGICAL EXAMPLE OF ORDINAL STRUCTURE

The typical psychological test of intellectual performance consists of a set of questions or problems (called "items" in the jargon of psychometrics). The person doing the test produces answers or solutions (called "responses"), and if these are evaluated as being either correct or incorrect the items are called "dichotomous." The standard procedure at this point is to count the number of correct

TABLE 2
A Summary of Ordinal Structures

A	STRICT SIMPLE ORDER transitive asymmetric connected	SIMPLE ORDER transitive antisymmetric strongly connected
B	STRICT PARTIAL ORDER WITH TRANSITIVE INDIFFERENCE	WEAK ORDER transitive strongly connected
C	STRICT PARTIAL ORDER transitive asymmetric	PARTIAL ORDER transitive antisymmetric reflexive

responses. This number is called the person's "total score" on the test. The total score is often converted to some kind of "transformed score" (e.g., an IQ or z score) for the purpose of enabling comparisons between peoples' performances on the test. In the following example we depart from this time–honored path to follow one pioneered by L. Guttman (1944). This bases comparisons on response patterns rather than on total scores.

Each person's performance on a set of dichotomous items may be described as a sequence of ones and/or zeroes. If the response to item i is correct then a "1" is placed in the ith position of the sequence; otherwise a "0" is placed in that position. This sequence of ones and/or zeroes describes the person's pattern of correct and incorrect responses. It is what we will call that person's "response pattern". If there are n dichotomous items in a test then the number of different possible response patterns is 2^n. For a three item test $2^n = 8$, and for this case the different possible response patterns are listed in Table 3.

Each response pattern describes a particular kind of performance on the test. Any two people having the same response pattern, have performed equivalently in the sense that they have responded correctly to precisely the same items. Any two people obtaining different response patterns, have performed differently in the sense that they have not responded correctly to the same items. This is true even when they have the same total score (e.g., if one's response pattern was 011 and the other's was 110). So the use of total scores as a basis for comparisons is not as informative as the use of response patterns.

Response patterns stand in a fairly obvious relation to one another. Each response pattern indicates a certain quality of performance. The quality of one response pattern may be superior to that of another in the following kind of way. A person who obtains response pattern 7 (in Table 3) has performed better than a person who obtains response pattern 5, simply because the first person not only responds correctly to every item that the second person gets correct, but also gets another item correct as well. By way of contrast, the performance of a person

TABLE 3
The Full Set of Eight Response Patterns Possible With Three Dichotomous Items

Response Patterns	Items 1	2	3
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1

who obtains the pattern 110 is not superior to the performance of a person who obtains the pattern 001, since neither is correct on any item correctly answered by the other. In terms of this relation they are indifferent. The relation involved here (let us call it "R_1") may be defined as follows: the quality of performance of a response pattern, i, is superior to the quality of performance of a response pattern, j, if and only if all items correct in j are correct in i and at least one item correct in i is not correct in j. The eight response patterns in Table 3 related by R_1 are shown in Fig. 5. In that figure a response pattern is superior to another if and only if an arrow points from it to the other.

It follows from the definition of this relation that it is transitive and asymmetric. Therefore, the set of all response patterns on an n item test related by R_1 is a strict partial order. Corresponding to this strict partial order there must be a partial order. This involves a slightly different relation (R_2) defined as follows: the quality of performance of a response pattern, i, is at least as good as the quality of performance of a response pattern, j, if and only if all items correct in j are correct in i. By definition, R_2 is transitive, antisymmetric and reflexive.

Does R_1 on the set of all possible response patterns for some test constitute either an interval order or a semiorder? The structure in Fig. 5 is a semiorder (and, hence, an interval order), so obviously, the set of all response patterns in a three item test related by R_1 constitutes a semiorder. However, for a four item test this is not so. The set of all possible response patterns for a four item test is listed in Table 4 and their structure according to R_1 is shown in Fig. 6. If this structure is an interval order, then any two ordered pairs of response patterns may be chosen, such as 8 and 6, and 15 and 11, and it must follow that either $R_1(8,11)$ or $R_1(15,6)$. Inspection of Fig. 6 shows that neither of these relations exist and, so, the set of all possible response patterns on a four item test is not an interval order and therefore not a semiorder.

Also, inspection of Fig. 5 shows that on a three item test, indifference with respect to R_1 is not transitive: response patterns 4 and 7 are indifferent, as are 4 and 5; but 7 and 5 are not. Hence, the set of all possible response patterns on a three item test related by R_1 is not a strict partial order with transitive indifference. Neither is R_2 strongly connected on this set. Therefore, related by R_2 it is not a weak order.

The fact that the set of all possible response patterns together with R_1 is not

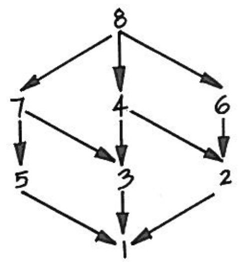

FIG. 5. A representation of the strict partial order imposed by R_1 on the response patterns in Table 3.

TABLE 4
The Full Set of Sixteen Response Patterns Possible With Four Dichotomous Items

		Items 1	2	3	4
Response Patterns	1	0	0	0	0
	2	0	0	0	1
	3	0	0	1	0
	4	0	0	1	1
	5	0	1	0	0
	6	0	1	0	1
	7	0	1	1	0
	8	0	1	1	1
	9	1	0	0	0
	10	1	0	0	1
	11	1	0	1	0
	12	1	0	1	1
	13	1	1	0	0
	14	1	1	0	1
	15	1	1	1	0
	16	1	1	1	1

even an interval order once the number of items exceeds three, explains why most mental test theorists have preferred total scores as the basis for any comparisons. Most current tests possess at least thirty items, making the strict partial order on the response patterns quite complex. This complexity reduces somewhat if the actual, rather than possible, response patterns are considered. Guttman's aim was to construct tests in which the set of actual response patterns ordered by R_1 was a strict simple order. This happens whenever the set of actual response patterns manifests a distinctive cumulative pattern. This pattern is illustrated in Table 5 for the case of the six item test.

If the set of actual response patterns includes nothing beyond these (although it need not include all of them) then R_1 on these patterns is a strict simple order. Such a set of response patterns is known as a Guttman scale within the

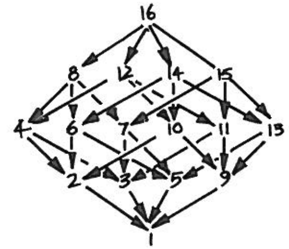

FIG. 6. A representation of the strict partial order imposed by R_1 upon the response patterns in Table 4.

TABLE 5
The Distinctive Cumulative Pattern Manifested by a Guttman Scale: A Set of Response Patterns Upon Which R_1 Imposes a Strict Simple Order

		Items 1	2	3	4	5	6
Response Patterns	1	0	0	0	0	0	0
	2	0	0	0	0	0	1
	3	0	0	0	0	1	1
	4	0	0	0	1	1	1
	5	0	0	1	1	1	1
	6	0	1	1	1	1	1
	7	1	1	1	1	1	1

psychometric literature. A Guttman scale is any set of response patterns simply ordered by R_2.

Guttman scales containing even a moderate number of items have proved difficult to construct. The consensus amongst mental test theorists is that they are impractical (e.g., the discussion by Nunnally, 1967). This view ignores the fact that our knowledge of the relevant attributes of test items is still quite poor. If the item attributes determining performance were known, then the possibility of constructing Guttman scales could be accurately assessed. In the absence of that knowledge the Guttman scale concept must not be dismissed, for Guttman scales make use of much more of the available information on test performance than does the use of total scores. In going from observed performance (i.e., response patterns) to total score, information is lost. Hence, the use of Guttman scales is much more efficient than the currently standard procedure.

The Guttman scale is not the only structure of interest in this context. If, for example, a set of actual response patterns related by R_1 is a strict partial order with transitive indifference then, by the process of reduction outlined above, a simple order would result. This is illustrated for a six item test in (a) in Fig. 7. Indifference with respect to R_1 occurs at three levels where there is a single indifferent pair. This makes indifference an equivalence relation (even if a degenerate one). The relation composed of the disjunction of R_1 and indifference weakly orders this set of response patterns. The relation in this case is: the quality of response pattern x is either superior or indifferent to the quality of response pattern y. The three equivalence classes consist of the three pairs of mutually indifferent response patterns. The response patterns within each pair are equivalent in the sense that whatever (in the set of actual response patterns) is superior or inferior to one is also superior or inferior to the other. So a simple order on classes of response patterns results.

(b) and (c) in Fig. 7 illustrate a semiorder and an interval order respectively for a six item test. As can be seen in these cases, the number of response patterns

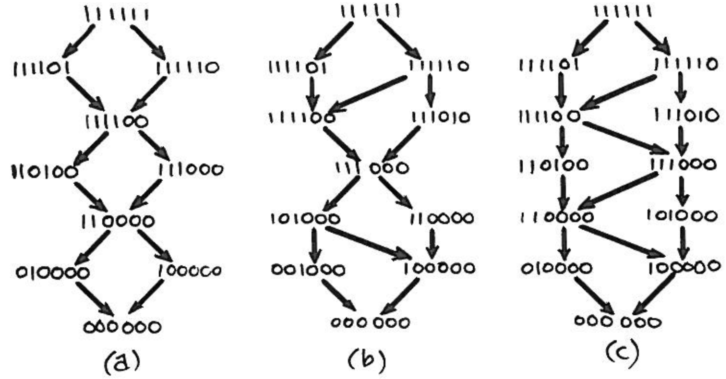

FIG. 7. An illustration of (a) a weak order, (b) a semiorder, and (c) an interval order for a six item test.

permitted progressively increases. A Guttman scale of six items allows only seven response patterns; an interval order permits twelve. In our present state of ignorance about the relevant item attributes these approximations to order may be more practical options than the Guttman scale itself.